



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

**FACULTY OF HEALTH, APPLIED SCIENCES, AND NATURAL
RESOURCES**

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	LEVEL: 5
COURSE: PROBABILITY THEORY 1	COURSE CODE: PBT501S
DATE: JANUARY 2023	SESSION: JANUARY
DURATION: 3 HOURS	MARKS: 100

SUPPLEMENTARY/ SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Dr. D. Ntirampeba
MODERATOR:	Mr. J. Amunyela

THIS QUESTION PAPER CONSISTS OF 5 PAGES
(Excluding this front page and statistical tables)

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programable calculator

ATTACHMENTS

1. Statistical tables (Z-Table)

QUESTION 1 [45 Marks]

1.1. Consider the events $A = \{1,3\}$, $B = \{2,5\}$, $C = \{6\}$, $D = \{3,4\}$ over a possibility space $S = \{1,2,3,4,5,6\}$. With reasons, state whether true or false:

1.1.1. Events A and C are mutually. [2]

1.1.2. $\{A, B, C, D\}$ is a partition of the sample space S [3]

1.1.3. $\emptyset^c = \{1,2,3,4,5\}$ [2]

1.1.4. $B \Delta C = \{1, 3\}$ [3]

1.2. Let X be a random variable with the probability mass function

$$p(x) = \begin{cases} \binom{5}{x} 0.4^x 0.6^{5-x} & x = 0,1,2,3,4,5 \\ = 0 & \text{otherwise} \end{cases}$$

1.2.1. Find, $F(x)$, the distribution function of the random variable X [4]

1.2.2. Find the median of X [2]

1.2.3. Find the mean of X [2]

1.3. A supplier of Kerosene has 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behaviour that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If Y denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modelled by

$$f(y) = \begin{cases} y, & 0 \leq y \leq 1 \\ 1, & 1 < y \leq 1.5 \\ 0, & \text{elsewhere} \end{cases}$$

1.3.1. Find $F(y)$ [5]

1.3.2. Find $p(0.5 \leq Y \leq 1.2)$ [5]

- 1.4. A large industrial firm purchase several new word-processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of wordprocessors, X , that are purchased each year has the following probability distribution:

X	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

If the cost on new wordprocessors at the end of this year is given by $12000 - 50X^2$,
in Namibia dollars,

- 1.4.1 how much can this firm expect to spend on new wordprocessors at the end of this year? [3]
- 1.4.2 find the variance of the number of wordprocessors that are purchased for this firm at the end of this year. [5]
- 1.4.3 find the coefficient of variation for the number of new wordprocessors that are purchased for this firm at the end of this year. [3]
- 1.5. If a dealer's profit, in units of N\$ 5000, on a new automobile can be looked upon as a random variable X having the density function defined by

$$f(x) = \begin{cases} 2(1 - x), & 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the:

- 1.5.1 average profit per automobile [4]
- 1.5.2 standard deviation of the profit per automobile [5]

QUESTION 2 [20 Marks]

2.1.

Factory workers are constantly encouraged to practice zero tolerance when it comes to accidents in factories. Accidents can occur because the working environment or conditions themselves are unsafe. On the other hand, accidents can occur due to carelessness or so-called human error. In addition, the worker's shift, 7:00 A.M.–3:00 P.M. (day shift), 3:00 P.M.–11:00 P.M. (evening shift), or 11:00 P.M.–7:00 A.M. (graveyard shift), may be a factor. During the last year, 300 accidents have occurred. The percentages of the accidents for the condition combinations are as follows.

Shift	Unsafe Conditions	Human Error
Day	5%	32%
Evening	6%	25%
Graveyard	2%	30%

If an accident report is selected randomly from the 300 reports,

2.1.1. what is the probability that the accident occurred on the graveyard shift? [2]

2.1.2. what is the probability that the accident occurred on either the evening or the graveyard shift? [2]

2.1.3. what is the probability that the accident occurred on either the evening or the graveyard shift given that there was a human error? [3]

2.2.

A diagnostic test for cancer is said to be 98% accurate if a person has the disease. Also, if a person does not have cancer, the test will report that he or she does not have it with probability 0.1. Only 0.1% has the disease in question. If a person is chosen at random from the population and diagnostic test indicates that he or she has cancer, what is the probability that he or she does, in fact, have cancer. [4]

2.3.

In a certain assembly plant, three machines, B_1 , B_2 , and B_3 , make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

- 2.3.1. What is the probability that it is defective? [4]
- 2.3.2. If a finished product is randomly selected and is found to be defective, what is the probability that it was made by machine B_2 ? [3]
- 2.3.3. If a finished product is randomly selected and is found to be defective, what is the probability that it was made by machines B_1 and B_2 ? [2]

QUESTION 3 [20 Marks]

- 3.1. University and college students average 7.2 hours of sleep per night, with a standard deviation of 40 minutes. If the amount of sleep is normally distributed,
- 3.1.1. What proportion of university and college students sleep for more than 8 hours? [3]
- 3.1.2. Find the amount of sleep that is exceeded by only 25% of students. [5]
- 3.2. The manager of a gas station has observed that the time required by drivers to fill their car's tank and pay are quite variable. In fact, times are exponentially distributed with mean 7.5 minutes. What is the probability that a car onset can complete a transaction in less than 5 minutes? [4]
- 3.3. The number of bank robberies that occur in large north American city is Poisson distributed with mean of 1.8 per day. Find the probability that between 10 and 12 (both values inclusive) robberies occur during a 5-day period. [3]
- 3.4. Major software manufacturers offer a help line that allows customers to call and receive assistance in solving their problems. However, because of the volume of calls, customers frequently are put on hold. One software manufacturer claims that only 20% of callers are put on hold. Suppose that 100 customers call,
- 3.4.1. what is the expected number of customers that are put on hold? [2]
- 3.4.2. what is the standard deviation of the number of customers that are put on hold? [3]

QUESTION 4[15 Marks]

4.1. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem find

$$P(5 < X < 15) \quad [5]$$

4.2

4.2.1. Let X be continuous uniform random variable on the interval $[a, b]$. show that

$$E(X) = \frac{a+b}{2} \quad [5]$$

4.2.2. A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a uniform distribution. What is the probability that an individual waits for more than 7 minutes? [5]

END OF EXAMINATION PAPER

Standard Normal Probabilities

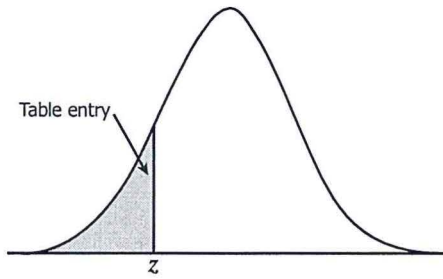


Table entry for z is the area under the standard normal curve to the left of z .

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

